§24. Spectrum of Subcritical Turbulence in an Inhomogeneous Plasma

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We study the slab plasma which is inhomogeneous in the z-direction, and the magnetic field is mainly in the y-direction. In order to analyze interchange mode turbulence, the relevant set of equations has been derived for the electrostatic potential ϕ, current $J_{\parallel}$ and pressure p; equation of motion, the Ohm's law and the energy balance equation, as [1]
$(\partial/\partial t)\nabla_{\perp}^2\phi + [\phi, \nabla_{\perp}^2\phi] = \nabla_{\parallel}J_{\parallel} + (\hat{y}\times\Omega')\cdot\nabla p + \mu_c\nabla_{\perp}^4\phi$
$\partial\Psi_{\parallel}/\partial t = -\nabla_{\parallel}\phi - \xi^{-1}(\partial J_{\parallel}/\partial t + [\phi, J_{\parallel}]) + \lambda_c\nabla_{\perp}^2 J_{\parallel}$
and $\partial p/\partial t + [\phi, p] = \chi_c\nabla_{\perp}^2 p$. The bracket [f,g] is the Poisson bracket, $[f, g] = (\nabla f\times\nabla g)\cdot\vec{b}$, ($\vec{b} = \vec{B}_0/B_0$,), Ω' is the average curvature of the magnetic field, $\Psi_{\parallel}$ is the vector potential, and the constant coefficient $1/\xi$ denotes the finite electron inertia, $1/\xi = (\delta/a)^2$, δ being the collisionless skin depth. Symbol $\parallel$ denotes the parallel to $\vec{b}$. The transport coefficients μ_c, λ_c and χ_c are the ion viscosity for the perpendicular momentum, the current diffusivity and the thermal diffusivity, respectively. The suffix c indicates the contributions from the collisional diffusion.

By the renormalization and by setting $\partial/\partial t = 0$, the nonlinear marginal stability condition is derived from the basic set of equations. The nonlinear dispersion relation for the existence of the nontrivial solution of the dressed test mode as

$$\left(\frac{\partial}{\partial k_z}\frac{k_x^2}{k_{\perp}^2}\frac{\partial}{\partial k_z} - \frac{\lambda_k\mu_k}{s^2}k_{\perp}^4 + \frac{G_0\lambda_k k_x^2}{s^2\chi_k k_{\perp}^2}\right)\phi_k = 0 \quad (1)$$

It is noted that the mode can be excited only if $G_0 = (dp_0/dz)\cdot(d\Omega/dz) > 0$. This is the reason G_0 is considered to be a driving parameter. This equation provides the nontrivial solution if the relation between eigenvalues μ_k, λ_k, χ_k is satisfied. We take $\phi_k = 0$ in the region $k < k_0$.

The dispersion relation was solved, and the k-dependencies of (μ_k, λ_k, χ_k) are derived. For fixed G_0, it is shown that the scaling relation

$$\mu_k(k),\ \lambda_k(k),\ \chi_k(k) \propto k_{\perp}^{-2} \quad (2)$$

is satisfied as an asymptotic nature. For the given ratios $k_x^2/k_{\perp}^2$ and $k_{\parallel}^2/k_{\perp}^2$, the equation (1) becomes independent of k if Eq.(2) is satisfied. Consequently, the ratios μ_k/χ_k and λ_k/χ_k become constant. The coefficient $\hat{C}_{\parallel}$ is introduced as $(k_{\parallel}/k_{\perp})^2 = \hat{C}_{\parallel}\lambda_k\mu_k k_{\perp}^4$. Noting that $\lambda_k\mu_k k_{\perp}^4$ is independent of k, the average of $\hat{C}_{\parallel}$, $\langle k_{\parallel}^2/k_{\perp}^2\rangle = C_{\parallel}\lambda_k(k_0)\mu_k(k_0)k_0^4$, is used. By use of the relations $\mu_k(k_{\perp}) = \mu_k(k_0)k_0^2 k_{\perp}^{-2}$ and $\chi_k(k_{\perp}) = \chi_k(k_0)k_0^2 k_{\perp}^{-2}$, the renormalized relations in [1] are reduced to an integral equation that determines the spectrum. It is approximately rewritten as

$$\mu_k(k_{\perp})\mu_k(k_0)k_0^2 = (1 + C_{\parallel} + G_0 w_a/\mu_k(k_0)\chi_k(k_0)k_0^4)^{-1}\int_{k_{\perp}}^{k_b} dk_{\perp}\, E(k_{\perp})$$

and so on. The parameter w_a characterizes the isotropy of the turbulence. (For the isotropic turbulence in the $\perp$-direction, $w_a = 1/2$ holds.) $C_{\parallel}$ is related to the ratio of nonlinear transfer rates, $P_r \equiv \mu_k/\chi_k = 1/\sqrt{1 + C_{\parallel}}$. Actual number of $C_{\parallel}$ is estimated and close to unity. The integral equation that $E(k_{\perp})$ must satisfy is finally given as

$$P_r\, w_a\, G_0\, k_{\perp}^{-2} = \frac{1}{2}\int_{k_{\perp}}^{k_b} dk_{\perp}\, E(k_{\perp}).$$

This integral equation is solved as

$$E(k_{\perp}) = 4\, P_r\, w_a\, G_0\, k_{\perp}^{-3} \quad (3)$$

in the energy containing region ($k_{\perp} << k_b$). The spectral function depends on G_0, and has the $k_{\perp}^{-3}$ dependence. The current spectral function is obtained as
$E_J(k_{\perp}) = \left[4w_a(1 - P_r^2)/\sqrt{P_r^2 + 1}\right]\xi G_0 k_{\perp}^{-3}$.
The pressure spectral function is also obtained as
$E_{\theta}(k_{\perp}) = 4w_a\,(dp_0/dz)^2\, k_{\perp}^{-3}$.

In summary, we presented the theory of the turbulence and turbulent-driven transport for subcritical turbulence far from the thermal equilibrium. The spectrum in the energy containing region is given by the power law.

References
[1] K. Itoh, S.-I. Itoh, A. Fukuyama, M. Yagi and M. Azumi, *Plasma Phys. Control. Fusion* **36**, 279 (1994) and **36** 1501 (1994).